# A stock rebuilding algorithm featuring risk assessment and an optimization strategy of single or multispecies fisheries 

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#### Abstract

Gröger, J. P., Rountree, R. A., Missong, M., and Rätz, H-J. 2007. A stock rebuilding algorithm featuring risk assessment and an optimization strategy of single or multispecies fisheries. - ICES Journal of Marine Science, 64. We present a simple but flexible stock-rebuilding algorithm model that features ideas of risk assessment, with all constraints set up explicitly, and with clear optimality for controlling fishing effort (or fishing mortality) and maximizing landings (or economic value). In contrast to the conventional approach, our approach does not predict future stock development from historical stock dynamics, but provides directly optimal annual $F$ values and associated optimum catch quotas for a given planning horizon. Hence, the $F$ values are not estimated retrospectively, but are realizations of a control variable created through the optimization process. The optimal solution is based on maximization of a non-linearly constrained objective function for catch or yield, whereas the constraints inter alia include biomass targets, $F$ limits, and stable catch. We present the basic theory together with selected model variants, such as inclusion of biological interactions and integration of elements of risk assessment. The optimization procedure outlined here is not only "risk averse" but a risk minimization procedure in itself. It can be applied in a deterministic or stochastic decision-making process as well as within a single or multispecies context. We illustrate the approach with a simplified (deterministic) multispecies fisheries management and a (stochastic) single-species risk assessment example.


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## Introduction

Finding better management strategies using multispecies or risk management approaches to support medium-term management decisions has become of increasing importance. Recently, the International Council for the Exploration of the Seas (ICES) has initiated several working groups and meetings, such as WKMIXMAN (Workshop on Simple Mixed Fisheries Management Models), SGMAS (Study Group on Management Strategies), or SGRAMA (Study Group on Risk Assessment and Management Advice), as well as the Symposium on Fisheries Management Strategies held in Galway, Ireland, in June 2006 to address this question (ICES, 2005, 2006a, b, c).

A key objective in fishery management is to maximize landings (or economic value) on a sustainable level. However, sustainability can be interpreted in different ways that are subject to different constraints. For instance, in the North Atlantic Fisheries Organization (NAFO) and the Northeast Atlantic Fisheries Commission (NEAFC) convention areas, the objective to maximize catch sustainably as part of the medium-term projection is constrained by the requirement to keep spawning-stock biomass (SSB) above some specified level. Therefore, managers have developed different mechanisms to maintain fisheries based on either effort or catch controls. Traditional management strategies are generally developed in equilibrium settings and relate to maximizing production, often based on a yield or catch-and-effort curve
whose maximum is interpreted as the most productive point of exploitation (maximum sustainable yield, MSY, with fishing mortality $F_{\mathrm{MSY}}$ ) (Hilborn and Walters, 1992). These techniques have been developed for single species (but see multispecies virtual population analysis, MSVPA) even though many fisheries impact many species simultaneously. Nevertheless, even in a single-species setting, the conventional techniques leave much to be desired because they do not always yield a clearly defined maximum. In addition, the theory that underlies these techniques is brought into question because there is no clear definition of constraints, most assumptions are implicit, and because they often achieve a poor fit to highly variable large fishery datasets.

At present, identification of optimal fishery management strategies is usually performed in four steps: (1) collecting population relevant data (commercial, market sampling and research survey data); (2) estimating relevant population parameters (stock abundances, fishing mortalities, etc.); (3) projecting and simulating future scenarios based on different management options and on the results of Step 2 (catch-effort or biomass-effort relationships); and (4) taking the most plausible result(s) of Step 3 as the optimal management strategy.

Currently, the commercial catch information used in Steps 1 and 2 is converted into population abundance using a population dynamics model and survey information. The estimated parameters are usually stock size in numbers and fishing mortality-at-age, but
depending on the type of model used, other parameters such as catchability can be estimated. This approach suffers from problems such as data inconsistency, limited temporal and spatial resolution, and the fact that environmental influences as well as technical and biological multispecies interactions are usually ignored. Another major problem is the difficulty of using retrospectively estimated parameters for future medium- to long-term management strategies. Moreover, medium-term scenarios are traditionally generated using analytical MSY approaches. These approaches are based on closed form logistic equations (such as the Schaefer model), rarely leaving space for explicitly including constraints, such as prespecifying key variables such as upper limits of effort or lower limits of biomass. However, accounting for such constraints turns out to be crucial in applied fishery management scenarios.

The approach presented here aims to overcome these deficiencies in traditional approaches. Combining elements of medium-term projection, multispecies and risk-management considerations, we draw on both operational research and econometric methods to create an adaptable framework for rebuilding stocks, given biomass targets and $F$ limits within a planning horizon set by managers. A numerical procedure using a non-linear optimization algorithm adopted from econometric control theory is introduced. This procedure can serve as a bridge between parameter estimation, scenario testing, forecasting, and risk assessment (i.e. between Steps 2 and 3 of the management procedures outlined above).

The idea of using aspects of dynamic programming in fisheries was first introduced by Rothschild (1972) in his treatise on defining fishery effort. Studies that more explicitly addressed the issue of maximizing (optimizing) a single-species harvest control rule soon followed (Walters, 1975; Hilborn, 1976). Since these early studies, other researchers have discussed harvest control rules and how to optimize them in a single-species setting, including a recent study by Quinn and Deriso (1999), who discussed detailed features of various types of objective function.

We further advance these studies by describing how to set up a general framework and optimize a multispecies harvest control rule that is subject to biological constraints. We show how a deterministic approach can be turned into a stochastic one, featuring aspects of risk assessment and management. As our approach does not "passively" predict future stock development based on parameter values estimated from the past, but rather "actively" seeks optimally to control it, the outcome provides both fishing mortality $F$ and catch values $C$, which can be used directly as recommendations for optimal fishing mortality and quotas or (total allowable catches) (TACs). We show that, because of its flexibility, our model is capable of easily incorporating other types of data, such as multispecies and environmental interactions, and economic considerations. We then illustrate how our model can be implemented by applying it to two examples: a quasi-realistic deterministic multispecies example based on a North American fishery located on Georges Bank, (GB), and a stochastic singlespecies example representing North Sea herring in ICES Subdivision IVb. Both examples are simplified to foster the understanding of the procedure. However, the software code used (we wrote the tools in SAS 9.1) can handle more complex situations.
horizon, in this case 10 years (2006-2015). The model starts with an initial multi-area, multispecies, age-disaggregated biomass in 2005, and ends with the target biomass in 2015. Although in principle the rebuilding period can be less than the planning horizon and may be variable among species, for simplicity, here we define them as equivalent and the same for all species. The initial biomass is the most recent biomass that was "estimated" by any traditional stock assessment method, e.g. ADAPT (Gavaris, 1990), statistical catch-at-age models (Quinn and Deriso, 1999), or Kalman filter (e.g. Harvey, 1989; Gudmundsson, 1994). The target biomass is the rebuilding biomass to be met at the end of the rebuilding period that is set by the fishery managers. For example, one might choose to use as target biomass $B_{\text {PA }}$. Starting with these initial conditions, annual biomass and fishing yields (controlled fishing) during the rebuilding period are followed. We control the fishing activity by setting upper limits for fishing effort or fishing mortality (e.g. $F_{\mathrm{MSY}}$ ).

Under this framework the optimal solution in terms of annual effort allocation (by management unit expressed as species, area, stock, or maybe as fleet, segment, or metiér) will be determined by maximizing total yield subject to the constraints. Fishery managers set the upper limits of fishing effort (or fishing mortality) and define targets for biomass rebuilding. Hence, the control or instrument variable is fishing effort $f$ (or fishing mortality $F$ ), and the objective function includes the total yield in physical or monetary units. Possible constraints are: " 1 t of haddock will have Xt of cod as bycatch, so the total catch of cod must be limited to 50 kg per trip"; and/or "by 2014, the SSB of cod must be equal to or larger than the rebuilding SSB target"; and/or "by 2007, the SSB of haddock must be equal to or larger than the rebuilding SSB target"; and/or "fluctuations in annual total catch should be minimal to ensure a relatively stable income for fishers".

Closed areas or seasons can simply be implemented as effort constraints by setting $F$ to 0 either constantly or temporarily in the area of concern. Therefore, the principle idea is to simulate scenarios and to iterate model parameters as long as these are non-optimal in terms of the optimization criterion.

## Model features

Here, we provide a detailed description of several model features, including the computation and maximization of physical and economic yield, stock size computation, incorporation of recruitment, biological and technical interactions, and conversion between fishing effort and mortality. For each model feature, we consider one species in a specific area that leads to the set of equations below. Note that, although all subsequently stated model equations could be easily extended and implemented using age, year, area, and species-disaggregated values and thus subscripts, for convenience and legibility, we suppress the subscripts for area in most cases except where necessary.

## Computation and maximization of physical yield

The central equation for calculating annual (physical) yield $Y_{k, a, y}$ per area for species $k$ at age $a$ in year $y$ is Baranov's catch equation (Baranov, 1918), which is multiplied by an estimate of


Figure 1. Graphical presentation of the rebuilding algorithm ( $D A S=$ days-at-sea).
weight-at-age, $W_{k, a, y}$ :

$$
\begin{equation*}
Y_{k, a, y}=C_{k, a, y} W_{k, a, y}=\frac{F_{k, a, y}^{*}}{Z_{k, a, y}^{*}} N_{k, a, y}\left(1-\mathrm{e}^{-Z_{k, a, y}^{*}}\right) W_{k, a, y}, \tag{1}
\end{equation*}
$$

where $N_{k, a, y}$ is the numbers at age $a$ in year $y$, the total mortality, $Z_{k, a, y}^{*}=F_{k, a, y}^{*}+M_{k, a}, M_{k, a}$ is natural mortality, fishing mortality $F_{k, a, y}^{*}=F_{k, y} S_{k, a}$, and $S_{k, a}$, which is selectivity at age $a$, can either be estimated empirically or specified by some selectivity function. Using a modified logistic function according to Bethke (2004), such as

$$
\begin{equation*}
S_{k, \text { length }}=\frac{1}{1+\mathrm{e}^{s_{k, 1}-s_{k, 2} \times \text { length }}} \tag{2}
\end{equation*}
$$

allows us to convert mesh size as a regulation measure into selectivity values. The model parameters $s_{k, 1}$ and $s_{k, 2}$ are estimated from prior experiments. The resulting $S_{k, \text { length }}$ values need to be further converted into age-based ones. Summing over ages and years for all species gives an estimate of total yield $Y$ that forms the major component of the objective function (optimization criterion) to be maximized. If the rebuilding period varies between species, the summation takes place over different time horizons. As we have to make sure that total biomass $B_{k}^{\text {total }}$ at the end of the rebuilding period matches or exceeds the target biomass $B_{k}^{\text {target }}$, this constraint is implemented via a penalty function (per species):

$$
\begin{equation*}
\text { Penalty term }{ }_{k}=\max \left(0, B_{k}^{\text {target }}-B_{k}^{\text {total }}\right) \tag{3}
\end{equation*}
$$

Biomass may be replaced by SSB. In the penalty term, we only penalize positive differences. This can be considered a further constraint
in addition to constraints on $f$ or $F$ values. The penalty term can be extended by multiplying it by a species-specific coefficient $\theta_{k}$ in order to weight some species more than others. Setting the elements of the coefficient matrix to 1 gives every species the same weight. The role of the coefficients is very important, because they ensure that the associated biomass targets will be met. The reason for this is that the magnitude of the penalty terms can vary by species, depending on the level of biomass and other conditions. Putting more weight onto a species implicitly means that the associated biomass constraint gains more weight in the objective function. Tuning the coefficients can help to balance the optimization between species. The objective function thus becomes

$$
\begin{equation*}
\text { Objective function }=Y-\sum_{k} \theta_{k} \text { Penalty term }{ }_{k} \tag{4}
\end{equation*}
$$

where total yield $Y$ will be maximized, whereas at the same time, the values of the species-related penalty terms are minimized.

In order to stabilize the expected annual catches (keeping the catch stable over time is more attractive for fishers because it keeps their income constant), the objective function can be optionally extended by introducing a smoothing term:

$$
\begin{equation*}
\text { Smoothing term }{ }_{k}=\sum_{y=1}^{T_{k}}\left(Y_{y, k}-Y_{k}^{\text {target }}\right)^{2}, \tag{5}
\end{equation*}
$$

where $T_{k}$ denotes the end of the rebuilding period for species $k$.
In contrast to the penalty term described above, we here penalize squared differences because we want to reduce the fluctuation in general. Again, species-specific weighting coefficients $\lambda_{k}$ could
be used. The objective function then becomes

$$
\begin{align*}
& \text { Objective function }= Y-\sum_{k} \theta_{k} \text { Penalty term } \\
& k  \tag{6}\\
&-\sum_{k} \lambda_{k} \text { Smoothing term } \\
& k
\end{align*} .
$$

Instead of an arbitrary yield target $Y_{y, k}^{\text {target }}$, average annual yield may be used as target in Equation (5), although this would increase the computer runtime somewhat because the average value will change during each iteration. Components of Equation (6) may be area-specific and summed to give the overall objective function.

## Computation and maximization of economic yield

An alternative objective function based on economic rather than physical yield can be derived following a similar procedure. Multiplying the physical yield in biomass ( kg ) by the speciesspecific unit price $P_{k, y}$ and totaling this up gives the total annual economic yield:

$$
\begin{equation*}
Y_{k}=\sum_{a} \sum_{y} Y_{k, a, y} P_{k, y} . \tag{7}
\end{equation*}
$$

Unit price may differ by species, but usually not by area. Depending on the species, the unit price may also vary by other factors, such as quality category or size group. Moreover, the unit price may change with the amount of fish landed (an economics rule of supply and demand). This may require the use of a feedback price function instead of a simple price function (interdependent or simultaneous price model) to calculate the unit price dependent on the amount of fish landed. As for the physical yield, we can add penalty and smoothing terms to the economic yield. If information on costs is available, the objective function might be modified by maximizing the profit as a criterion, instead of income and turnover, respectively.

## Stock dynamics

Equation (1) contains stock numbers-at-age whose dynamics are modelled as

$$
\begin{equation*}
N_{k, a, y}=N_{k, a-1, y-1} \times\left(1-\mathrm{e}^{-Z_{k, a, y}^{*}}\right) ; \text { for } 1<a<\operatorname{age}_{k}^{\max } . \tag{8}
\end{equation*}
$$

$N_{k, 1, y}$ is calculated as recruitment of the preceding year either using a density-dependent or -independent stock-recruitment function; in the case of density-dependence, we use the Ricker approach (Ricker, 1954):

$$
\begin{equation*}
N_{k, 1, y+1}=R_{k, y}=R_{k, 1} \operatorname{SSB}_{k, y} \mathrm{e}^{-R_{k, 2} \operatorname{SSB}_{k, y}} \tag{9}
\end{equation*}
$$

with SSB and model parameters $R_{2}$ and $R_{\mathrm{k}, 2}$.
In the case of weaker density-dependence, we use the Beverton-Holt approach (Beverton and Holt, 1957):

$$
\begin{equation*}
N_{k, 1, y+1}=R_{k, y}=\frac{\mathrm{SSB}_{k, y}}{\mathrm{SSB}_{k, y}+g R_{k}^{\max }} R_{k}^{\max } \tag{10}
\end{equation*}
$$

with $g$ the slope and $R_{k}^{\max }$ the asymptote of the stock-recruit relationship. In both cases, simple linearizations exist. If recruitment shows some dependence on environmental factors, we use an extended recruitment function (Hilborn and Walters, 1992).

Therefore, for the Ricker curve with environmental factor $E_{\gamma}$

$$
\begin{equation*}
\ln \left(\frac{R_{k, y}}{\operatorname{SSB}_{k, y}}\right)=\ln \left(R_{k, 1}\right)-R_{k, 2} \mathrm{SSB}_{k, y}+c E_{y}, \tag{11}
\end{equation*}
$$

where $c$ is a regression coefficient, and $R_{k, 1}$ and $R_{k, 2}$ have the same meaning as above.

The recruitment functions given above can easily be replaced by other types of recruitment function, or simply by conditional vectors of discrete empirical values. As we use a segmented regression approach (a so-called "hockey stick" function) in our second example on risk assessment of North Sea herring, a brief description will be given below, together with some references.

The $\mathrm{SSB}_{k, a, y}$ will be calculated taking into account the agespecific maturity Mat ${ }_{k, a}$ and weight pattern $W_{k, a}$ :

$$
\begin{equation*}
\operatorname{SSB}_{k, a, y}=N_{k, a, y} W_{k, a} \operatorname{Mat}_{k, a} . \tag{12}
\end{equation*}
$$

Species interactions can be addressed in different ways, dependent on whether we consider technical (e.g. bycatch issues) or biological interactions (e.g. predator-prey interactions). One way to incorporate technical interactions is to use a simple bycatch matrix $C_{k, a}^{\text {bycatch }}$ containing values of observed proportions (ratios per target species) of caught species sorted by target species in the fishery. If at the same time we take into account the age-specific selection pattern $S_{k, a}$, this leads to the following re-formulation of fishing mortality:

$$
\begin{equation*}
F_{k, a, y}^{*}=F_{k, y} S_{k, a} C_{k, a}^{\text {bycatch }} \tag{13}
\end{equation*}
$$

Similarly, predation on recruits can be incorporated in a number of different ways, for example, using a linearized version of the Ricker stock-recruit relationship for a predator of age $a$ feeding on recruits (Hilborn and Walters, 1992):

$$
\begin{align*}
\ln \left(\frac{R_{y, \text { prey }}}{\operatorname{SSB}_{y, \text { prey }}}\right)= & \ln \left(R_{1}\right)-R_{2} \times \mathrm{SSB}_{y, \text { prey }} \\
& +c(\text { predator density })_{a} \tag{14}
\end{align*}
$$

(the species subscript $k$ has been dropped for ease of notation). Incorporating predator-prey relationships at a population level at a later stage is not difficult in principle; it can, for example, be done by splitting natural mortality into a predation and a residual component:

$$
\begin{equation*}
M_{a}=M_{a, \text { predation }}+M_{a, \text { residual }} . \tag{15}
\end{equation*}
$$

The real difficulty arises from the question of how to estimate the natural mortality components. In order to do so, one could consider correlations in species' occurrence, or as in the case of the traditional MSVPA, stomach contents, consumption rates, etc.

The conversion of fishing effort $f$ into fishing mortality $F$ is another important issue. The reason for this is that, in contrast to fishing effort $f$, which is the actual variable to be controlled and therefore used by fishery managers, fishing mortality $F$ is the decision variable, i.e. the parameter optimized in the model. We therefore have to take into account catchability $q_{k, a, y}$, which
is the interfacing coefficient between both quantities:

$$
\begin{equation*}
F_{k, a, y}=q_{k, a, y} f_{k, y} . \tag{16}
\end{equation*}
$$

## Implementing stochasticity, bias, and ideas of risk assessment

We believe that risk assessment and management will become increasingly important in fisheries assessment and management over the next few years. This can be seen inter alia from the fact that ICES most recently initiated a study group on Risk Assessment and Management Advice (SGRAMA; ICES, 2006c).

The procedures described here represent one way that some of these issues can be approached. Input values such as initial abundance values, weights, maturity observations, and recruitment are prone to error. Errors in general create uncertainty, and uncertainty creates risk. The errors can be of systematic (bias) or random (stochastic) nature, and can address, for instance, the fact that initial abundance values might have been overestimated, or that recruitment varies randomly with some variance around the deterministic/estimated function chosen.

Systematic bias can be implemented easily by multiplying the relevant quantities by some bias factor, for instance 0.8 , to address the fact that this quantity's input is only $80 \%$ in size and not $100 \%$. Similarly, the so-called implementation error, which is an error addressing the fact that fishers may spend more effort or catch more fish than stipulated, can be incorporated by using higher values of $F$ to calculate future stock dynamics than were generated by the optimization procedure. As an example, suppose we have an annual $20 \%$ implementation error, we would then multiply the optimal $F$ values by 1.2 and use the modified $F$ values to calculate the next year's stock dynamics.

In contrast to bias, randomness can be either added to the recruitment model as an additive or multiplicative error term with an appropriate distribution function [see for instance, Equations (9) and (10)]. This converts the deterministic approach into a stochastic one. Unless the errors are iid, the error structure may include autocorrelation and/or heteroscedasticity, and therefore can become rather complicated (see our North Sea herring example below; Fogarty, 1993; Power, 1996; Jiao et al., 2004). Re-running the random procedure many times would result in many different outcomes. This could possibly lead to violations of the biological constraints, because the SSB limit set may be undercut.

In general, such undercuts can be interpreted as negative (hazardous, harmful) events and will happen with some frequency, in probabilistic terms with some likelihood. In risk assessment terminology, risk may be defined as

$$
\begin{equation*}
\text { Risk }=P(\text { harmful event }) \times \text { severity of harmful event }, \tag{17}
\end{equation*}
$$

which is risk in a juridical sense. This definition of risk is not only implemented as part of many national constitutions (for instance, of the German constitution; Schulte, 1999), it is also commonly used in engineering, in natural or environmental sciences, or in medicine (Burgmann, 2005). In mathematical sciences, however, the probability of a harmful event is often solely used as a definition for risk. As we aim at specifying costs or loss from a political and economic perspective, Equation (17) turns out to be the appropriate risk measure, because it contains a probability term specifying
the chance or likelihood of a harmful event as well as a severity term quantifying the magnitude of the loss. Therefore, we would not merely have to count how often the SSB limit set was undercut, but also what was the severity of undercutting the SSB limit (magnitude of consequence, effect size, costs, loss function, etc.). In most cases, that is not easy to define and derive, respectively. It can be done either in physical or monetary terms, and in terms of an immediate or a future effect. In the context of the discussion above, one option to interpret this would be to calculate the physical loss of SSB that may affect future reproduction and therefore the recruitment leading to the following risk definition:

$$
\begin{align*}
\text { Risk }= & P(\text { lower SSB limit undercut }) \\
& \times \text { expected loss of SSB } \\
= & P^{L} \times L^{e} . \tag{18}
\end{align*}
$$

The SSB lower limit may be adopted from the precautionary approach and taken as $\mathrm{SSB}_{\mathrm{PA}}$ or $\mathrm{SSB}_{\text {lim }}$. To simplify the notation, we do not distinguish between alternative definitions and therefore use SSB ${ }^{\text {target }}$ as an equivalent for SSB lower limit. The expected loss may be derived from the difference between realized SSB and SSB ${ }^{\text {target }}$. In statistical notation, the expected loss $L^{e}$ turns out to be a conditional expectation:

$$
\begin{equation*}
L^{e}=E\left[\text { SSB }^{\text {target }}-\mathrm{SSB} \mid \mathrm{SSB}<\mathrm{SSB}^{\text {target }}\right] . \tag{19}
\end{equation*}
$$

In the simulation part of the optimization process, SSB is replaced by the simulated values, $\widehat{\mathrm{SSB}}$, and empirical mean values are used as estimates for the (unobservable) expectations. Suppose that in $n^{(-)}$ out of $n$ cases, $\widehat{\mathrm{SSB}}$ falls short of $\mathrm{SSB}^{\text {target }}$. Then $P\left(\mathrm{SSB}<\mathrm{SSB}^{\text {target }}\right)$ can be estimated by $\hat{P}^{L}=n^{(-)} / n$, and expected loss by $\hat{L}^{e}=$ $\overline{\text { SSB }^{\text {target }}-\widehat{\operatorname{SSB}}}{ }^{(-)}$, where the superscript $(-)$denotes that averaging is restricted to the $n^{(-)}$cases with SSB $<$SSB $^{\text {target. }}$. Hence, $\widehat{\text { Risk }}=$ $\hat{P}^{L} \times \hat{L}^{e}$, or, equivalently,

$$
\begin{equation*}
\widehat{\text { Risk }}=\overline{\max \left(0, \mathrm{SSB}^{\text {target }}-\widehat{\mathrm{SSB}}\right)} \tag{20}
\end{equation*}
$$

In this notation, "empirical" risk relates to the penalty term (3) in objective functions (4) and (6), respectively. Hence, the optimization outlined here is in itself a risk-minimizing procedure, a very important feature for managers. During the simulation procedure, confidence limits encompassing the risk trajectory may be calculated using (empirical) quantiles of SSB ${ }^{\text {target }}-\widehat{S S B}$.

Further information on the theory underlying risk assessment and risk management can be found in Francis and Shotton (1997), Lane and Stephenson (1997), and Burgmann (2005). For a formal treatment of quantitative risk assessment and management, see McNeil et al. (2005).

## Some numerical aspects

## Dynamic programming and the planning horizon

Numerically, we deal with a dynamic programming problem in discrete time where we use a constraint for the control variable $F$ together with penalties for the state of SSB and Y. In contrast to classical dynamic programming problems (see for instance, Bertsekas, 2001, 2005) which typically use a 1 year or infinite planning horizon (rebuilding period), we are interested in a fixed planning duration of several years to decades. The reason for this is that it is biologically (but also economically) neither realistic
nor feasible to believe that a stock currently at very low biomass can abruptly jump above a desired high biomass level (the rebuilding biomass) within 1 year. Hence, depending on the biological and economic constraints, we must accept that it usually takes a longer rebuilding time to bring a specific stock "smoothly" above a certain minimum biomass level. Consequently, the most plausible strategy would be to install the control at the end of the planning horizon, because the performance of the entire rebuilding period needs to be measured, and also because the target biomass is the level of biomass that needs to be reached at the end of the rebuilding period, and not before. Hence, in our fisheries applications, it does not make sense to set up an annual penalty term. According to Bertsekas (2005), this means that any feedback, i.e. evaluation of the objective function, will be "postponed until the last possible moment" of the planning horizon at whose end the learning process linked to the fish stock dynamics will be completed. Consequently, as a basis for our decision, we need to postpone the control until all effects have been accumulated over the time of the planning horizon. Summing up the effects assures that the next feedback-loop iteration will be based on a complete knowledge update. As long as the solution is suboptimal, the algorithm continues iteratively looping by selecting new values of $F$ for the time unit and calculating the objective function at the end of the entire planning horizon until it finds an optimal solution (i.e. ideally the global maximum of the objective function). The algorithm ends its looping by applying a pre-defined numerical stopping rule.

Our approach is strictly identical to classical theory in the special case of a 1 year rebuilding period. However, although we are interested in setting up the feedback based on the information accumulated over the entire planning horizon, to portray the real world we need to specify and implement the non-linear stock dynamics on a year-by-year basis if the planning horizon is longer than 1 year and if we possess this information per annum. Within the planning horizon, therefore, the non-linear translation of a previous year's production into the next year is done using the set of dynamic equations outlined above. This may be interpreted as feed-forward propagation between years, but within the planning horizon.

Our rebuilding framework can be implemented numerically using algorithms based on methods of non-linear optimization. A large set of different algorithms with different requirements can be found in the literature on numerical mathematics, and many of these have been implemented in various programs, such as MATLAB and SAS (Statistical Analysis Software; SAS Institute Inc., 1999).

During the search process, an iterative process is used to return the objective function's value for each iterated alternative. This iterative approach is sometimes called simulation, so the entire algorithm can also be called simulation-based optimization (Azadivar, 1992). It is usually necessary to initialize the algorithms with starting $F$ (or $f$ ) values. For solving the equations, we used the optimization algorithms (e.g. the Dual-Quasi-Newton Optimization Algorithm, which functions without the need to specify derivatives) implemented in SAS/IML version 9.1.3 (SAS Institute Inc., 1999), because of its ability to manipulate large-scale matrices while simultaneously allowing the simulation to be embedded into a macro-based statistical environment, making it possible to vary options and to carry out advanced statistical calculations.

## Optimization

The dual-quasi-Newton algorithm used here functions on a heuristic basis, meaning it does not use total enumeration to find the optimal solution, but rather some alternative time-saving "search" strategy.

An evaluation of the SAS routines clearly goes beyond the scope of this paper. Instead, we decided to accept the solution of SAS in a first step, and in a second step performed a search procedure in the neighbourhood of the optimized $F$ value to substantiate the assumption of a global optimum. The search grid used covers a reasonable range of values of $F$, and the procedure is detailed below.

We use species-specific multipliers for the penalty terms [see equation (6)], which have been a priori fixed on the basis of previous knowledge. Fixing these parameters instead of estimating the weights together with the control variable $F$ allows us to circumvent the problem of the "curse of dimensionality" (Hillier and Lieberman, 2004) induced by too high a number of parameters estimated when optimizing both the control variables and the associated multipliers. Apart from this, and for factual reasons, it would not make much sense to estimate the weights.

## An illustrative deterministic multispecies example

## Input data

In this example, we draw on real data collected from a groundfish fishery on GB and in the adjacent Gulf of Maine (GOM). This relatively simple example is used for illustrative purposes to show some of the model features; it is not intended to discuss thoroughly the GB groundfish fishery or to criticize and replace any of the existing assessment results and/or management strategies.

We limit our consideration to five species, of which one is split into two components (stocks). The species and stocks chosen include the haddock, Melanogrammus aeglefinus (GB stock), yellowtail flounder, Pleuronectes ferrugineus (GB stock), witch flounder, Glyptocephalus cynoglossus (GOM and GB combined stock), American plaice, Hippoglossoides platessoides (GOM and GB combined stock), and Atlantic cod, Gadus morhua (separate GOM and GB stocks). All species/stocks are assessed and managed by the National Marine Fisheries Service (NMFS), for which GB and GOM witch flounder and American plaice are each managed as one stock. On the other hand, the two cod stocks (GB and GOM) are assessed and managed separately by NMFS; for computational reasons, we consider them as two different species in order to allow simplified implementation of the bycatch matrix presented below. Most of the relevant stock data were taken as reported in the Groundfish Assessment Review Meeting (GARM) for 2005 (NEFSC, 2005).

We base our scenarios on a planning horizon (rebuilding period) of 10 years, starting with 2006. All data used are age-resolved and consist of abundance estimates, weight, partial recruitment, and maturity observations. The abundance estimates are based on VPA estimates derived from domestic commercial catch data and scientific surveys. Data related to weight, maturity, and partial recruitment stem either from market sampling or from previous technical experiments, and were provided by NMFS researchers (S. X. Cadrin, pers. comm.).

The bycatch data used are taken from an industry-based survey performed by the School of Marine Science and Technology (SMAST) (University of Massachusetts, New Bedford, MA, USA), mainly on GB. The data were aggregated over 3 years to stabilize results. These bycatch data are used to set up the matrix of technical interactions [Equation (17)] (Table 1). Cell entries consist of normalized fractions of bycatch per bycatch species (column) and for each target fishery (row); therefore, its diagonal contains exclusively values of 1 ; its off-diagonal values are greater than zero if there are technical interactions and zero if there are not. We used the same interaction matrix for all 10 years in the planning horizon.

Table 1. Matrix of technical interactions among species and stocks used in the optimization model illustration for the GB groundfish fishery, based on the data derived from a triennial industry-based trawl survey as well as from a multi-annual cod tagging experiment in the GOM area.

| Target fishery | Bycatch species |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\operatorname{Cod}(\mathrm{GB})$ | Cod (GOM) | Haddock | Yellowtail flounder | Witch flounder | American plaice |
| Cod (GB) | 1 | 0.04 | 0.13 | 0.04 | 0.07 | 0.03 |
| Cod (GOM) | 0.01 | 1 | 0.13 | 0 | 0.07 | 0.03 |
| Haddock | 0.26 | 0.01 | 1 | 0.05 | 0.07 | 0.07 |
| Yellowtail flounder | 0.12 | 0.01 | 0.04 | 1 | 0.02 | 0.03 |
| Witch flounder | 0.48 | 0.02 | 0.06 | 0.03 | 1 | 0.14 |
| American plaice | 0.16 | 0.01 | 0.01 | 0.02 | 0.34 | 1 |

As we consider two cod stocks, we have to relax the usual definition of a technical interaction, which is applied normally to different species, and apply it to two different stocks. As we usually cannot distinguish between individuals of different populations or stocks of one species in the catch, we utilized the results of a cod tagging programme (http://www.gmamapping.org/ codmapping/20893) to provide us with estimates. This allowed us to infer a percentage of $1 \%$ cod movement from the GOM area into the GB area, and of $4 \%$ cod movement the other way, which we took as bycatch estimates. As we have no data on technical interactions between cod and other species in the GOM area, we assumed the same values as for the GB area.

We took estimated versions of recruitment functions and other parameter values from the GARM report (NEFSC, 2005) (Table 2). The SSB rebuilding targets (in tonnes) to be reached at the end of the 10 -year rebuilding period are defined here as species-specific estimates of biomass reference points ( $B_{\mathrm{MSY}}$ ). The lower $F$ limit for the optimization process is set to 0 ; the upper $F$ limit not to be exceeded during the optimization process is represented by estimates of $F_{\text {MSY }}$. The values for natural mortality $M$ are 0.2 for all species/stocks except for witch flounder, which is 0.15 .

The objective function used here maximizes the overall catch at the end of the rebuilding period, and is constrained by rebuilding targets in the following manner:

$$
\begin{align*}
\text { Objective function } & =C_{\text {total }} \\
& -9.0 \max \left(0,\left[216780-\mathrm{SSB}_{\mathrm{GB} \text { Cod }}\right]\right) \\
& -2.0 \max \left(0,\left[82830-\mathrm{SSB}_{\mathrm{GOM} \text { Cod }}\right]\right) \\
& -3.2 \max \left(0,\left[250300-\mathrm{SSB}_{\text {Haddock }}\right]\right) \\
& -3.7 \max \left(0,\left[58800-\mathrm{SSB}_{\text {Yellowtail Flounder }}\right]\right) \\
& -10.0 \max \left(0,\left[25248-\mathrm{SSB}_{\text {Winter Flounder }}\right]\right) \\
& -1.0 \max \left(0,\left[28600-\mathrm{SSB}_{\text {American Plaice }}\right] .\right. \tag{21}
\end{align*}
$$

The weighting factors are chosen so that the rebuilding target biomass will be reached for certain at the end of the rebuilding period. These coefficients must be adjusted iteratively so that the biomass values do not undercut the associated biomass targets.

Our strategy of optimizing the $F$ values is chosen to be in compliance with the NMFS strategy of "constant $F$ values" (S. X. Cadrin, pers. comm.), i.e. for each species, we optimized only one $F$ value (instead of a set of annual values simultaneously) and kept this value constant over the entire rebuilding period. For comparison, we also did the calculations based on the "flexible F strategy".

## Results and interpretation

The dual-quasi-Newton optimization algorithm terminated with a feasible solution for the estimated/optimized parameters ( $F$ values). As this does not necessarily mean that the global maximum was found, we profiled the objective function by systematically changing the optimized $F$ values within a broad range. The results showed that no other maximum of the objective function could be found within a bandwidth of $0-20 \times F^{\text {optimized }}$, with increments of 0.01 , neither in the constant case nor the flexible strategy case (Figure 2). The profile peaks when the $F$ multiplier is 1 , meaning that, based on this criterion, the global maximum was within the range given and indicated by the dashed vertical line in Figure 2. However, we should keep in mind that we used a heuristic strategy and that only full enumeration would give $100 \%$ certainty to have found the global optimum. The final objective function value was given as 514833 (rounded), which is basically the value of the total catch in tonnes accumulated in time minus the six biomass constraints (which in this case all yielded 0 , because all goals were met). The accumulated catch numbers per species were: $123913 \mathrm{t}(\operatorname{cod} G B)$,

Table 2. Recruitment functions, lower biomass targets, and upper F limits by species for the GB groundfish fishery example, based on the data given in NEFSC (2005).

| Species/stock | Recruitment | Rebuilding target SSB ${ }^{\text {target }}$ ( $\mathbf{t}$ ) | Upper $F$ limit ( $F_{\text {MSY }}$ ) |
| :---: | :---: | :---: | :---: |
| Cod (GB) | $R=58569.90$ SSB /(182 $740.90+\mathrm{SSB}$ ) | 216780 | 0.175 |
| Cod (GOM) | $R=9854.365 S B /(7516.10+$ SSB $)$ | 82830 | 0.225 |
| Haddock | If SSB $<75000 \mathrm{t}$, then $R=9879$ else $R=10615$ | 250300 | 0.263 |
| Yellowtail flounder | If SSB $<5000 \mathrm{t}$, then $R=13220$ else $R=24444$ | 58800 | 0.25 |
| Witch flounder | Mean $R=32549.5$ | 25248 | 0.23 |
| American plaice | Mean $R=8813$ | 28600 | 0.166 |

[^0]

Figure 2. Value of objective function for a range of levels of the $F$ multiplier. The dashed vertical lines indicate the position of the optimal $F$ value.

104400 t (cod GOM), 247968 t (haddock), 19815 t (yellowtail flounder), 8355 t (witch flounder), and 10399 t (American plaice).

Biological results from all five species (and six stocks) over the 10 -year rebuilding period are shown in Figure 3. Plotted are time-series of SSB $, B, C$, and optimized $F$ values, as well as their associated limits. All rebuilding targets in terms of SSB were met, and all total $F$ values were kept below the given limits. With the exception of haddock, in all cases $B$ and SSB differed from each other substantially. For haddock, the similarity of the $B$ and SSB curves were a result of the early maturation strategy of haddock. Haddock seem to be also a special case from another point of view: even keeping the $F$ values constant at a very low level led to a dramatic decrease in the two biomass levels, so that the general trend for both biomass curves was persistently negative. Re-running the optimization program under the same conditions but with a no-bycatch assumption did not change the pattern for haddock. From this, we would infer that the upper $F$ limit stated in the last GARM report (NMFS, 2005) was too high for haddock, because our result was much smaller ( 0.07 instead of 0.26). In the case of the no-bycatch scenario, the two $F$ curves lay close together for haddock, and a somewhat higher overall catch ( 548001 t ) was obtained because of a higher allocation of allowed catches, in particular for American plaice.

Although this example was designed to focus on the "constant $F$ value strategy", we also wanted to compare the results with that of a "flexible $F$ value strategy". This strategy allows annually changing $F$ values to be selected. A global maximum was also found (Figure 2, vertical dashed line), so all estimated $F$ values were optimal based on the given maximization criteria. The comparison
of "constant $F$ " and "flexible $F$ " strategies (Table 3, Figure 4) showed that the latter reached a higher overall catch at the end of the rebuilding period $(+45886 \mathrm{t}=+8.9 \%)$ as well as higher individual catches for all species/stocks except American plaice. Looking at the species-disaggregated catches indicated that specifically the allowed haddock and yellowtail flounder catches became substantially larger than those under the "constant $F$ strategy" and therefore could be significantly optimized. As an example using yellowtail flounder, Figure 4 illustrates that the constant $F$ strategy kept the optimal annual catches (quotas, TACs) stable whereas the flexible $F$ strategy allowed more variation in annual catches. The patter was the same for haddock and American plaice. In the cases of the two cod and single witch flounder stocks, the catches showed similarly low fluctuations for the flexible $F$ and constant $F$ strategies.

We also performed the "flexible $F$ value strategy" based on a no-bycatch assumption. As before under the "constant $F$ strategy", this scenario led to a further increase in allowed total catch (637 277 t ), equivalent to a $12 \%$ rise, mainly through an allocation of bigger haddock, yellowtail flounder, witch flounder, and American plaice catches. The steepest increase was for American plaice, yielding a catch four times higher than with the bycatch scenario. This was due to higher $F$ values being directly and exclusively allocated to individual target fisheries during the optimization process, because these were uncorrelated under the no-bycatch assumption.

To evaluate the influence of a different bycatch matrix, we carried out a simple experiment. Bycatch fractions were multiplied in $50 \%$ of the cell entries of the interaction matrix for the three species GOM cod, yellowtail flounder, and witch flounder by factors $0.5,0.8,1.0,1.2,1.5$, and 2 , respectively. The experiment showed that there was some influence of bycatch size leading to slight changes in individual optimal quotas. Over the entire rebuilding period, this influence was also reflected in the overall catches, with a maximum change of $10 \%$ between the original situation (factor 1.0) and the situation with doubled bycatch (factor 2.0), and a minimum change of $0.2 \%$ for the three species (factor 0.5). However, this influence made it necessary to re-adjust the weighting coefficient of the penalty term in only three cases (of 30 ).

## Single-species risk assessment example with a stochastic recruitment function and implementation error <br> Input data

In order to demonstrate the stochastic and risk assessment features of our framework, we applied our approach to a single-species case. We therefore ignore interactions with other species, but integrate specific uncertainties in the input data.

Risk is understood here to be the probability of undercutting a SSB limit set by ICES times the loss of the associated SSB [Equation (18)]. We use data derived from a herring fishery located in Subdivision IVb of the North Sea. This herring stock is assessed by ICES and managed by the EU. Most of the relevant stock data were taken from ICES (2006d). The 2005 input data for this stock (age $0-9$ abundance, weight in kg , selectivity, maturity, M, upper limit $F_{\text {MSY }}$ ) are given in Table 4. The SSB rebuilding target set by ICES is 1300 t . The ICES recommendations for $F_{\text {MSY }}$ are 0.12 for juveniles and 0.25 for adults. Because the risk for $F_{\mathrm{MSY}}=F_{\mathrm{PA}}=0.25$ is considered to be quite low, in order to


Figure 3. Trajectories for biomass (B, SSB), fishing mortality ( $F$ ), recruitment $R$ (left panels), and optimal catch and change in catch (right panels). (a) GB cod; (b) GOM cod; (c) haddock; (d) yellowtail flounder; (e) witch flounder; and (f) American plaice.

Table 3. Optimal catches for the multispecies example for constant $F$ and flexible $F$ strategies.

| Species/stock | Constant $\boldsymbol{F}$ <br> strategy | Flexible F <br> strategy | \% difference |
| :--- | :---: | :---: | :---: |
| Cod (GB) | 123913 | 132005 | 6.53 |
| Cod (GOM) | 104400 | 105022 | 0.6 |
| Haddock | 247968 | 277646 | 11.97 |
| Yellowtail flounder | 19815 | 27846 | 40.53 |
| Witch flounder | 8355 | 8853 | 5.96 |
| American plaice | 10399 | 9363 | -9.96 |
| Total | 514849 | 560735 | 8.91 |

make our example risk assessment results more illustrative we used a higher arbitrary $F_{\text {PA }}$ value of 0.45 for adults. We also added a $20 \%$ implementation error by increasing $F$ by $20 \%$ over the value of $F_{\text {optimized }}$ calculated during the optimization process for the current year (i.e. 2005). The conditions were chosen to simulate a hidden and stronger exploitation of adult herring than given by the optimal $F$ value. All other starting values were kept as reported in ICES (2006d).

Again, we base our scenarios on a planning horizon (rebuilding period) of 10 years, starting with 2006. However, the objective function used here is simpler than in the multispecies case and maximizes the overall herring catch at the end of the rebuilding


Figure 4. Comparison of biomass ( $B$ and SSB), fishing mortality ( $F$ ), and catch trajectories (TAC) for constant (left panels) and flexible $F$ strategies (right panels) for yellowtail flounder.

Table 4. North Sea herring data from the central North Sea (Subdivision IVb) taken from ICES (2006) used for the stochastic risk assessment example.

| Age | Abundance (numbers) | Weight (kg) | Selectivity | Maturity | M | $F_{\text {MSY }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 19.620 E9 | 0.01 | 0.1155 | 0 | 1 | 0.15 |
| 1 | 7.230 E9 | 0.041 | 0.2145 | 0 | 1 | 0.15 |
| 2 | 5.140 E9 | 0.11 | 0.4217 | 0.7 | 0.3 | 0.25 |
| 3 | 7.510 E9 | 0.162 | 0.7459 | 0.65 | 0.2 | 0.25 |
| 4 | 2.140 E9 | 0.207 | 1 | 1 | 0.1 | 0.25 |
| 5 | 2.410 E9 | 0.223 | 1.0757 | 1 | 0.1 | 0.25 |
| 6 | 0.590 E9 | 0.248 | 1.0305 | 1 | 0.1 | 0.25 |
| 7 | 0.280 E9 | 0.271 | 1.0688 | 1 | 0.1 | 0.25 |
| 8 | 0.290 E9 | 0.285 | 1 | 1 | 0.1 | 0.25 |
| 9 | 0.090 E9 | 0.289 | 1 | 1 | 0.1 | 0.25 |

period. It is constrained in the following manner:

$$
\begin{align*}
\text { Objective function }= & C_{\text {total }}-\max (0,[1.3 E 6-\mathrm{SSB}]) \\
& -0.01(1-0.15) \max \left(0,\left[C_{y}-\bar{C}\right]^{2}\right) . \tag{22}
\end{align*}
$$

The third term with weight 0.01 serves to stabilize the annual catches with mean (catch) being the average catch during the rebuilding period. The 0.01 weighting value has pre-experimentally been chosen to maintain catch variations approximately within $\pm 15 \%$ (which is the current catch variation allowance set by the European Commission) and at the same time to ensure that the algorithm will converge. Although in this example, the allowance term is set to 0.15 (to reflect the $\pm 15 \% \mathrm{EC}$ rule), in principle, its value can range between 0 and 1 .

As recruitment function, we used the "hockey stick" function currently in use at ICES for the stock (ICES, 2006d), which is based on estimates from a segmented regression and to which we added a noise term $\theta_{y}$ based on a log-normal assumption with first order autocorrelation. Therefore, the complete recruitment function becomes

$$
R_{y}= \begin{cases}49.34 E 9 \times e^{\theta_{y}} & \text { if } S S B_{y-1} \geq 537 E 6,  \tag{23}\\ 49.34 E 9 \times \frac{S B_{y-1}}{537 E 6} \times \bar{e}^{\theta y} & \text { if } S S B_{y-1}<537 E 6\end{cases}
$$

with

$$
\begin{equation*}
\theta_{y}=0.4561 \times \theta_{y-1}+\epsilon_{y}, \tag{24}
\end{equation*}
$$

in which, $\theta_{y}$ is a normally distributed error with mean 0 and variance 0.572 (ICES, 2006d). The autoregression parameter was estimated based on the recruitment residuals of the segmented regression. To simulate the autocorrelated noise, we used a function implemented in SAS, basically generating $\theta y$ from a normal distribution by making sure that all simulated values were independent, and inserting this into Equation (24), and this then into [Equation (23)].

## Results and interpretation

Both recruitment and SSB fluctuated greatly between years (Figure 5a). The vertical lines indicate the maximum and minimum values reached. The trajectories for the optimal $F$ values did not exceed the upper $F$ limit (Figure 5b); the vertical lines indicate the annual minima and maxima of the simulated $F$ values. However, the $F$ curve associated with the implementation error lay $20 \%$ above the optimal $F$ curve. From the year 2008 on, it always exceeded the upper $F$ limit, because the implementation error is a component that cannot be controlled. Estimated catch


Figure 5. Risk assessment results ( 1000 runs) for North Sea herring: (a) dynamics of recruitment and SSB, (b) optimum and impaired fishing mortality, (c) optimum catch (TAC) and catch change (TAC change), and (d) $P\left(S S B<S S B_{P A}\right)$, SSB loss, and risk. The continuous or dashed lines connect the means. The vertical bars indicate the range between minimum and maximum values reached during the stochastic optimization process.
values which are associated with optimal $F$ values can be considered as quota or TAC recommendations (Figure 5c). On average, these catches appeared to be relatively stable and stayed generally within the upper and lower $15 \%$ limits.

Counting the frequency of failures and deriving the differences between the SSB and the lower SSB limit (SSB ${ }^{\text {target }}$ ) indicated the magnitude of annual risk (Figure 5c). The probability and the severity term, and therefore the risk, began to increase continuously from 2008 on, reaching a maximum in 2015. Table 5 summarizes the realizations of the three quantities of concern $\left[\hat{P}^{L}=\right.$ $\hat{P}\left(\right.$ SSB $<$ SSB $\left.^{\text {target }}\right)$, severity $\hat{L}^{e}$ and $\widehat{\text { Risk }] ~ i n ~ t h e ~ f i r s t ~ t w o ~ t a b l e ~}$ rows. Most importantly, the second row shows that the rebuilding constraint was not met at the end of the planning horizon by indicating a high risk of 1649.20.

Comparing the rows of Table 5, we can see that, even with the higher arbitrary $F_{\mathrm{PA}}$ of 0.45 for adults and a white noise error term added to the recruitment function, the risk appeared to be quite low (mean risk for the entire rebuilding period: 0.62 , row 5 , last year's risk: 2.33 , row 6 ). However, adding either an implementation error of $20 \%$ or a first-order autocorrelation to the error term, or both, caused the risk to increase dramatically (Table 5, rows $1-4$ ).

To examine the influence of overestimated initial abundance, we additionally simulated two cases with either $50 \%$ or $80 \%$ of initial numbers. For this experiment, we took the baseline $F$

Table 5. Comparison of three risk scenarios for the herring example: (1) with autocorrelation and with implementation error,
(2) without autocorrelation but with implementation error, and (3) base case. The average situation of the rebuilding period and that of the last (i.e. rebuilding) year were compared for the three scenarios.

| Time-frame | Scenario | $\hat{\mathbf{P}}\left(\mathbf{S S B}<\right.$ SSB $^{\text {target }}$ ) | SSB loss | $\widehat{\text { Risk }}$ |
| :---: | :---: | :---: | :---: | :---: |
| Entire period (averaged) | (1) | 0.0460 | 6366 | 292.84 |
| Last year |  | 0.1022 | 16137 | 1649.20 |
| Entire period (averaged) | (2) | 0.0148 | 1255 | 18.57 |
| Last year |  | 0.0319 | 3127 | 99.75 |
| Entire period (averaged) | (3) | 0.0031 | 199 | 0.62 |
| Last year |  | 0.0063 | 369 | 2.33 |

value of 0.25 and switched off all other biases except the randomness of the recruitment model. Only for the $50 \%$ reduction case did SSB fall below the lower SSB limit during the second year (i.e. 2007), but it recovered thereafter (not shown). It appears, therefore, that the optimization algorithm can compensate effectively for such cases.

## Discussion

There is a tremendous amount of literature regarding management issues with focus, for instance, on setting specific harvest control rules (Punt and Smith,1999; Smith et al., 1999; Cochrane, 2002; Roel et al., 2004; Campbell and Dowling, 2005; Johnston and Butterworth, 2005; Punt et al., 2005), on establishing specific recovery or rebuilding plans (Kelly et al., 2006), on the integration of uncertainty (Hilborn, 1985; Butterworth and Bergh, 1993; Cochrane et al., 1998; Butterworth and Punt, 1999; Dichmont et al., 2005), on integrating fleet and fisher behaviour (Vestergaard, 1996; Vignaux, 1996; Campbell and Hand, 1999; Holland and Sutinen, 1999; Wilen et al., 2002; Baelde, 2003; Hutton et al., 2003), on the incorporation of economic aspects (Bockstael and Opaluch, 1983; Dupont, 1993; Sampson, 1994; Mardle and Pascoe, 1999; Holland, 2000; Holland and Sutinen, 2000; Mistiaen and Strand, 2000; Smith and Wilen, 2003), and finally on the integration of environmental effects (De Oliveira et al., 2005). However, apart from the studies of Babcock and Pikitch (2000), De Oliveira and Butterworth (2004), and Lewy and Vinther (2004), most concentrate on single-species procedures, and none addresses the problem in the same manner using the same combination of elements as we have done.

Drawing on optimal control methods, we have proposed a simulation approach for sustainable fishery management, explicitly stating a control variable (here fishing mortality or fishing effort) that will be optimized during the process within a priori specified upper and lower limits. It does this by maximizing an objective function that explicitly allows incorporation of various constraints. These may refer to the species, stocks/populations, and areas incorporated; the type of the $R-\mathrm{SSB}$ relationship used, the length of the planning horizon, the penalty terms, the lower and upper size limits of effort or fishing mortality, the maximization criterion (physical catch, monetary catch, profit, etc.), the weighting of the components in the maximization criterion, and the risks. In particular, in multispecies applications, technical interactions can be accounted for by specifying an appropriate bycatch matrix. This matrix can be extended to incorporate fleets, segments, metiérs, or other management units considered necessary by the European Commission. It can also make use of spatial or tagging information.

Instead of basing the model on a simple annual biomass propagation at a stock level, we explicitly used a regeneration function approach, accounting for recent biological knowledge on stock recruitment. However, any method that relies on a regeneration function based on reproduction is limited by our understanding of stock recruitment. To remedy this, we have two options:
(i) To replace the age-disaggregated analytic model by some holistic production model that ignores recruitment using, for instance, the following annual biomass propagation at a stock level:

$$
\begin{align*}
\text { Biomass }_{\text {current year }}= & \text { Biomass }_{\text {last year }}+\text { Growth }_{\text {last year }} \\
& - \text { Catch }_{\text {last year }} . \tag{25}
\end{align*}
$$

In a multispecies context, we can also mix one or more production functions associated with one or more stocks with age-disaggregated equations linked to other stocks. However, we should keep in mind that the use of holistic models is always a trade-off, because such models have
often been criticized as being too simplistic because they ignore essential biological features.
(ii) The other option is to use an age-disaggregated model with a recruitment function explicitly incorporated, and to extend the model by including uncertainty in an appropriate way, as we have tried to do here. Moreover, allowing for uncertainty/stochasticity in the regeneration function ensures that ideas of risk assessment can be easily integrated into the model.

## Specific results of the deterministic multispecies example

Our examination of the two optimization examples shows that the type of framework proposed in this paper is extremely flexible and can handle constant or flexible $F$ management strategies, stochasticity, risk assessment, and multispecies or single-species cases.

Contrasting the constant $F$ strategy with the flexible $F$ strategy showed that the idea of keeping the optimized $F$ values constant during the planning horizon, to stabilize the catches, is comparable with the flexible $F$ strategy when the objective function of the flexible $F$ strategy is used with the catch stabilization term. The flexible $F$ strategy is therefore equivalent to the European strategy (though quota-controlled management is used), where the change in next year's TAC should not differ by more than $15 \%$ of the previous year's TAC. The results in terms of catch and biomass in our simulation were of the same magnitude for the two approaches. According to S. X. Cadrin (pers. comm.), a constant $F$ value strategy might have the advantage of enhancing effort control. In contrast, a flexible $F$ strategy, despite allowing higher catches, may not be desirable to fishers because of the resulting bigger variance in annual catches (even though, for instance for yellowtail flounder, some $41 \%$ more catch/income would be realized, Table 5).

In our examples, selecting the weight values as part of the objective function required some prior experimentation, because the terms to be minimized (i.e. the differences between the speciesspecific biomass targets and the related estimated biomasses) were of different magnitude and therefore had a different weight within the objective function. As this is not simply linear (inter alia because of interacting bycatch effects), it takes some time to find the right tuning. Correct adjustment of the weights means that none of the species-related biomass targets will be undercut. Finding the correct biomass weights becomes more difficult when conflicting options are used. Obviously, the flexible F strategy can handle this situation more easily than a constant $F$ strategy, because there are more alternatives to choose annual $F$ values and therefore to balance out shifts in the dynamics.

Because fish density may have an effect on the likelihood of being taken as bycatch, we carried out some numerical experiments by changing the values of the bycatch matrix, which showed that the final results are not that much affected (i.e. the algorithm still came up with an optimal solution that was close to the case without changes in the bycatch matrix). This finding may have to do with the standardization of the bycatch values and the fact that the bycatch matrix is not directly used to manipulate the catch, but indirectly as an exponential multiplier for $F$ [Equation (17)]. However, in rare cases these experiments also showed that re-tuning the weights associated with the biomass targets as part of the objective function may be necessary to
ensure that the estimated biomass will not fall below the biomass target. To incorporate bycatch variation or time trends, one option would be to link the bycatch ratio to the annual abundance estimated during the optimization process in cases where this relationship is well understood. As we aggregated our bycatch data over a 3 -year period to stabilize the results, another option may be to use the disaggregated data and therefore the variability to make the bycatch matrix random.

## Specific results of the stochastic single-species example

Risk is something that is generated by uncertainty in the underlying data. However, as a statistical quantity for decision-making, risk is helpful for managers to use as an indicator and to decide upon acceptance or rejection of a management strategy. As part of this framework, risk is something that is implicitly minimized [Equations (3) and (18) or (20)], so risk assessment is one component that is automatically supported by our rebuilding framework. Using monetary units would help to create more complex objective functions and to change the rebuilding framework into a more general ecosystem management approach integrating a variety of competing ecosystem elements and constraints to find the point of optimum in ecosystem use.

We have demonstrated that our optimization framework addresses many of the issues raised by the SGRAMA (see above). We have also shown that it is very important to learn more about sources of errors and their effects on the dynamics and magnitude of risk by manipulating options in optimization runs. For instance, autocorrelation can have a trend-amplifying effect, periodically driving the recruitment up and down. In our single-species example, reducing the magnitude of exploitation by either reducing the upper $F$ limit or switching off the implementation error or the autocorrelation feature results in an annual risk of falling below the biomass target that is close to zero. Even at higher $F$ limits, the risk is negligibly small. This means that these hidden mechanisms can have a strong negative effect on the dynamics of some populations and need to be considered by managers.

Underestimating start abundances does not appear to have a large effect on the optimization results, because the algorithm compensates for this bias.

## Implications of EU fishing regulations

Note that the approach presented here is appropriate in accounting for any kind of (linear) restriction in the optimization algorithm. Restrictions can be implemented irrespective of the underlying reasoning behind the restriction. This means that restrictions need not necessarily be based on scientific (biological or economic) considerations, but may reflect political or regulatory restraints. This feature turns out to be of particular importance: on the one hand, any optimization approach incapable of including politically motivated restraints would be of limited value for real (EU) fishing applications. On the other hand, an approach accounting for such constraints allows a clearcut analysis of the constraints' impacts on the result of the optimization procedure.

In our application examples, we pointed out the particular role of the $15 \%$ rule concerning TAC changes in the EU. This regulation serves as a "stability constraint", because it aims at guaranteeing a stable income of fishers. However, it turned out to conflict seriously with the constraint to stay above the biomass target at
the end of the rebuilding period. Even substantial adjustment of the "objective function weights" associated with the individual biomass target constraints may not be able to compensate for the TAC changes. In those situations, it may be necessary to allow a relaxation of the $15 \%$ maximum for TAC changes and to reduce the quota to achieve the desired biomass target.

## Prospects for future research

We consider our stock-rebuilding approach to be an adaptive dynamic framework still subject to improvement, because our knowledge of the biological, technical, and economic mechanisms of fisheries is increasing. The modular structure of the approach allows for incorporation of additional information by an appropriate modification of the optimization model equations. In reality, we feel that improvements of the basic approach presented here are most likely to result from more detailed experiments, surveys, or analysis of existing data concerning the following key issues:
(i) interactions
(a) biological
(1) analysis and modelling of multispecies interactions based on the food of groundfish species;
(b) technical
(1) development of better methods to quantify bycatch and discards,
(2) modelling the impact of discards on the GB foodweb;
(c) environmental
(1) analysis and modelling of species-environment interactions.
(ii) analysis and modelling
(a) of stock-recruitment relationships and their interlink to other biotic and abiotic factors.
(iii) fish distribution
(a) analysis of fish distribution patterns as they relate to habitat and environmental factors and how distribution affects stock assessment,
(b) analysis of regional migratory patterns.
(iv) more detailed interpretation/analysis of hydroacoustic data.

Apart from knowing whether this biological knowledge is available or not, the concept of the rebuilding framework in principle incorporates modifiable functions, vectors, and matrices addressing all the points above ("place-holders"). The required information must "only" be determined and then inserted. For instance, if we have more detailed information on migration patterns from digital storage tags and geolocation methods (Gröger et al., 2007), the spatial component could be easily implemented by either introducing a migration matrix or by modifying the matrix of technical interactions (i.e. the bycatch matrix). Or, if we have better information about predator-prey relationships using, for instance, stomach content investigations, we might be able to set-up a matrix of biological interactions similar to the bycatch matrix, but where the columns are represented by the
prey species and the rows by the predator species. This matrix could then be multiplied with an $M$ vector (a vector of either assumed or given "default" or "residual" natural mortality) in a similar fashion to what we have done with the bycatch matrix and the optimized $F$ vector using Equation (13).

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[^0]:    $R$ is for age 1 in thousands, except for witch flounder (age 3).

